

Interpolating solution from AdS_5 to hyperscaling violating Lifshitz space-time

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Abstract

We construct two interpolating solutions in type II string theory which interpolate between an AdS_5 in the UV and a hyperscaling violating three (spatial) dimensional Lifshitz space-time in the IR. The first solution is non-supersymmetric and is obtained from a known intersecting non-supersymmetric D3-brane with chargeless D0-brane solution of type IIB string theory, by restricting some parameters characterizing the solution and going to a new coordinate. In the IR the dilaton is non-constant in general and the metric is three (spatial) dimensional hyperscaling violating Lifshitz with dynamical critical exponent $z = (3 + 3\gamma)/(3 - \gamma)$ and hyperscaling violation exponent $\theta = 12/(3 - \gamma)$, where γ is a real parameter and can take continuous values from -1 to $+1$. At the two extreme values, i.e., for $\gamma = \pm 1$, the dilaton is constant. On physical grounds it is found that γ must be restricted to lie in the range $-1 \leq \gamma \leq 0$. The second solution is supersymmetric and is obtained from the known F-D2 bound state solution of type IIA string theory by zooming into a particular region of space. In the UV, the proper description is given in a T-dual frame which is AdS_5 , whereas in the IR, it gives a three dimensional hyperscaling violating Lifshitz with $z = 3$ and $\theta = 2 = d - 1$.

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1. Introduction : Conventional theoretical tools are inadequate to give explanations of the physics behind the strongly interacting quantum many body systems. Holography plays a crucial role in dealing with such systems by mapping them to some gravitational theories in one dimension higher. This goes by the name AdS/CFT correspondence [1] or more generally gauge/gravity duality [2] where a strongly coupled field theory in D dimensions is dual to a weakly coupled gravity theory in $D + 1$ dimensions. This duality has broad applications not only for relativistic systems like QCD [3], but also for the non-relativistic strongly interacting condensed matter systems [4] and can be used to understand the physics behind such systems. A particular class of latter systems which has attracted a lot of interests in recent times are those having Lifshitz and Lifshitz-like (with hyperscaling violation) scaling symmetries at their quantum critical point. Properties of these systems can be understood by holographically constructing their gravity duals which show the same scaling symmetry as an isometry (upto a conformal transformation).

In addition to space-time translation and spatial rotation, Lifshitz symmetry is a non-relativistic scaling symmetry where space and time scale differently $t \rightarrow \lambda^z t$ and $x^i \rightarrow \lambda x^i$ ($i = 1, 2, \dots, d$, with d being the spatial dimension of the theory, λ , the scaling parameter and $z(\neq 1)$, the dynamical critical exponent). For $d = 2$, the gravitational theory whose solution (metric) exhibits the above scaling symmetry as an isometry has been given in [5, 6]. Similar metrics have also been shown to arise from different solutions of string theory, (gauged) supergravity in [7]. When systems have Lifshitz-like scaling symmetry, the physical quantities do not scale as in the case of Lifshitz symmetry, but they scale as if the dimension of the system is reduced by a parameter called the hyperscaling violation parameter (θ) [8]. In the corresponding gravitational theory the solution (or the metric) is not invariant under the scaling we mentioned, but changes upto a conformal factor $ds \rightarrow \lambda^{\theta/d} ds$ [9, 10, 11, 12]. These metrics have also been shown to arise from various string theory and (gauged) supergravity solutions in [13, 14]. A class of systems belonging to this group satisfying $\theta = d - 1$ is particularly interesting as they give logarithmic violation of the area law of the entanglement entropy [11] which in turn indicates the presence of a hidden fermi surface in the system [12].

In the previous works [13] we have shown how space-time having Lifshitz-like symmetry arises from various supersymmetric string theory solutions. In this paper, among other things, we will show how similar symmetry can arise from some non-supersymmetric string theory solution³. However, Lifshitz and Lifshitz-like symmetries being non-relativistic sym-

³These solutions in both asymptotically flat as well as non-flat spaces were originally constructed in arbitrary dimensions in [15] (see also [16]). Various properties of these solutions, including BPS limits, brane-antibrane interpretations and open and closed string tachyon condensations were studied in [17, 18]. Similar non-supersymmetric brane-like solutions in asymptotically $\text{AdS}_5 \times S^5$ spaces have been constructed in [19, 20, 21, 22]. However, no stability analysis has been performed for these non-supersymmetric solutions. The main reason could be that these solutions have naked singularities and might be called unphysical, however, their status in full string theory is not at all clearly understood. For extended solutions of the type we are

metries we expect them to appear at low energy or at IR of the field theory and this implies by holography that it should appear when the inverse radial coordinate $r \rightarrow \infty$ in the corresponding gravity theory. On the other hand, when we go to UV in the field theory, i.e., when we take $r \rightarrow 0$ in the gravity side we expect that the full relativistic symmetry will be restored. This in turn implies that there must exist interpolating solutions which will interpolate between relativistic solution like AdS_5 in the UV ($r \rightarrow 0$) and some non-relativistic solution like Lifshitz or Lifshitz-like solution in the IR ($r \rightarrow \infty$). Indeed such solutions have already been reported in the literature [25, 26, 27, 28, 29, 30, 31, 32]. In [25], some non-supersymmetric string theory solution interpolating between AdS_5 and an anisotropic Lifshitz has been constructed. In [27], various interpolating solutions of the type mentioned have been obtained in D dimensional AdS gravity with massive vector field and also in $\mathcal{N} = 4$, $D = 6$ gauged supergravity theory. More recently, a supersymmetric string theory solution interpolating between AdS_5 and a Lifshitz with hyperscaling violation with $z = 7$ and logarithmically running dilaton has been obtained [30]. Different non-supersymmetric solutions interpolating between Bianchi attractor in the IR and AdS or Lifshitz in the UV have been constructed in [31].

In this paper we construct both a non-supersymmetric and a supersymmetric interpolating solutions in type II string theory which interpolate between AdS_5 in the UV and a Lifshitz with hyperscaling violation in the IR. For the non-supersymmetric case, We start from a known non-supersymmetric D3-brane intersecting with chargeless D0-brane solution of type IIB string theory [33, 18]. This solution is characterized by four independent parameters. By going to a suitable coordinate, fixing one of the parameters and then zooming into a suitable region we construct the solution which gives AdS_5 in the UV ($r \rightarrow 0$) and a one parameter family of Lifshitz with hyperscaling violation in the IR ($r \rightarrow \infty$). For the hyperscaling violating Lifshitz solution the dynamical critical exponent has the value $z = (3 + 3\gamma)/(3 - \gamma)$ and the hyperscaling violation exponent has the value $\theta = 12/(3 - \gamma)$, where γ is a real parameter which takes continuous values in the range $-1 \leq \gamma \leq 1$. However, from the form of the energy momentum tensor of the boundary theory, we find that γ must lie in the range $-1 \leq \gamma \leq 0$, such that the energy and the pressure densities of the solution remain positive semidefinite. The dynamical critical exponent, therefore, lies in the range $0 \leq z \leq 1$ and the hyperscaling violation exponent lies in the range $3 \leq \theta \leq 4$. The spatial dimension of the theory is $d = 3$. The dilaton in general is not constant, but it becomes constant only at the two extremes when $\gamma = \pm 1$. On the boundary the interpolating solution can be interpreted as a flow from $\mathcal{N} = 4$, $D = 4$ super Yang-Mills theory to a non-relativistic many body theory in three spatial dimensions having a hyperscaling

going to discuss and also for asymptotically AdS solutions the dynamical stability [23] is usually correlated with the thermodynamic stability [24] of the solutions. As we will discuss later, even the thermodynamic stability is not helpful to conclude the stability of the solutions considered here.

violating Lifshitz symmetry.

For the supersymmetric case, on the other hand, we start from a known 1/4 supersymmetric F-D2 bound state solution of type IIA string theory. By zooming into a particular region of space for the F-D2 solution we construct the interpolating solution which interpolates between AdS_5 in the UV and a $d = 3$ hyperscaling violating Lifshitz space-time in the IR with $z = 3$ and $\theta = 2$. Actually in the UV, the interpolating solution of F-D2 gives $\text{AdS}_4 \times \mathbb{R} \times \text{S}^5$, with one of the directions (representing \mathbb{R}) getting smaller in size and should be compactified. In that case the solution becomes nine dimensional and in string frame is given by $\text{AdS}_4 \times \text{S}^5$, with non-zero dilaton. However, if we want to remain in ten dimensions, the above supergravity solution is not good as it can not be trusted when the radius of the circle corresponding to the compact direction becomes small compared to the string length. The proper supergravity description then would be given by its T-dual configuration which can be seen to be precisely $\text{AdS}_5 \times \text{S}^5$.

2. The interpolating solutions : This section consists of three subsections. In the first subsection we give the construction of interpolating solution from some non-supersymmetric D3-brane solution of type IIB string theory. In the second subsection we try to interpret the interpolation as a flow in the boundary theory. In the third subsection we describe the construction of interpolating solution from some supersymmetric F-D2 bound state solution of type IIA string theory.

2.1 Interpolation from non-supersymmetric solution: To construct the interpolating solution we begin with certain intersecting non-supersymmetric solution of type II string theory discussed in [33, 18]. In [18], we have constructed non-supersymmetric Dp -brane intersecting with chargeless D1-brane and D0-brane solution (given in Eqs.(4) – (6) of that paper) for the purpose of showing an interpolation between black Dp -brane and KK bubble of nothing and interpreting the interpolation as closed string tachyon condensation⁴. We will use not exactly this solution but a variant of it so that there is no D1-brane present. This is achieved by setting the coefficient of $(dx^1)^2$ and that of $(dx^i)^2$ for $i = 2, 3, \dots, p$ equal. That means we put $\delta_0 = \delta_2$ in that solution. Furthermore, we consider $p = 3$, i.e., it represents non-supersymmetric D3-brane intersecting with chargeless D0-brane of type IIB string theory.

⁴We remark that the interpolation from black hole to bubble in gravity solutions occurs when we change some of the parameters of the solution continuously by hand from one set of values to another. So, the interpolation occurs through a continuous series of singular solutions. However, this interpolation can be interpreted as a transition from black hole to bubble by closed string tachyon condensation only under certain conditions and not in general [18]. The solution we deal with here does not interpolate between black hole and bubble and so it cannot be interpreted as a transition under any condition.

The solution can be easily read off from Eqs.(4) – (6) in [18] and can be written as⁵,

$$\begin{aligned}
ds^2 &= -F^{-\frac{1}{2}}(\rho) \left(\frac{H(\rho)}{\tilde{H}(\rho)} \right)^{\frac{3\delta_1}{8} + \frac{3}{2}\delta_0} dt^2 + F^{-\frac{1}{2}}(\rho) \left(\frac{H(\rho)}{\tilde{H}(\rho)} \right)^{-\frac{5\delta_1}{8} - \frac{1}{2}\delta_0} \sum_{i=1}^3 (dx^i)^2 \\
&\quad + F^{\frac{1}{2}}(\rho) (H(\rho) \tilde{H}(\rho))^{\frac{1}{2}} \left(\frac{H(\rho)}{\tilde{H}(\rho)} \right)^{\frac{3\delta_1}{8}} (d\rho^2 + \rho^2 d\Omega_5^2) \\
e^{2(\phi - \phi_0)} &= \left(\frac{H(\rho)}{\tilde{H}(\rho)} \right)^{\frac{\delta_1}{2} - 6\delta_0}, \quad F_5 = (1 + *)Q \text{Vol}(\Omega_5)
\end{aligned} \tag{1}$$

where the various functions appearing in the above solution are defined as,

$$\begin{aligned}
H(\rho) &= 1 + \frac{\omega^4}{\rho^4}, \quad \tilde{H}(\rho) = 1 - \frac{\omega^4}{\rho^4} \\
F(\rho) &= \left(\frac{H(\rho)}{\tilde{H}(\rho)} \right)^\alpha \cosh^2 \tilde{\theta} - \left(\frac{\tilde{H}(\rho)}{H(\rho)} \right)^\beta \sinh^2 \tilde{\theta}.
\end{aligned} \tag{2}$$

In (1) the metric is given in the Einstein frame, ϕ is the dilaton field and ϕ_0 is its asymptotic value. F_5 is the self-dual five form, where Q is the charge parameter and $*$ denotes the Hodge dual. The solution is characterized by seven parameters, namely, α , β , δ_0 , δ_1 , ω , $\tilde{\theta}$, and Q . The equations of motion give us three relations among them and they are

$$\begin{aligned}
\alpha - \beta &= -\frac{3}{2}\delta_1 \\
\frac{1}{2}\delta_1^2 + \frac{1}{2}\alpha(\alpha + \frac{3}{2}\delta_1) + \frac{3}{2}\delta_0^2 &= (1 - 3\delta_0^2)\frac{5}{4} \\
Q &= 4\omega^4(\alpha + \beta) \sinh 2\tilde{\theta}.
\end{aligned} \tag{3}$$

Therefore, there are actually four independent parameters in the solution⁶. Also note that the solution has a singularity at $\rho = \omega$ and so, the physical region is given by $\rho > \omega$. We now make a change of coordinate by the following relation,

$$\rho = \bar{r} \left(\frac{1 + \sqrt{g(\bar{r})}}{2} \right)^{\frac{1}{2}}, \quad \text{where, } g(\bar{r}) = 1 + \frac{4\omega^4}{\bar{r}^4} \equiv 1 + \frac{r_0^4}{\bar{r}^4} \tag{4}$$

⁵It is possible to take a double Wick rotation of this solution by taking $\rho \rightarrow it$ and $t \rightarrow -ix^4$ to obtain a real, time dependent, space-like D3-brane solution which is anisotropic in x^4 -direction. In a special coordinate system this solution has recently been shown [34] to give a four dimensional de Sitter space upto a conformal transformation upon compactification on some six dimensional product space.

⁶Here one might think that the solution contains too many parameters in contradiction with the Birkhoff's theorem. However, we point out that such uniqueness theorem applies for regular manifolds and not for singular manifolds as is the case here. The physical interpretations of these parameters are not well-understood. In one interpretation they can be related to number of D3 branes, number of anti-D3 branes, number of D0 branes and a tachyon parameter [33]. Note that since the solution has non-zero D3 brane charge, the number of D3 branes and number of anti-D3 branes are different and their difference gives the total D3 brane charge. On the other hand the solution has no D0 brane charge, as this is a non-BPS D0 brane. We remark that although this is a solution of type IIB string theory, it contains a D0 brane, where this D0 brane is non-BPS.

and by this the singularity at $\rho = \omega$ has now been shifted to $\bar{r} = 0$. In terms of this new coordinate various functions given in (2) take the forms,

$$\begin{aligned} H(\rho) &= 1 + \frac{\omega^4}{\rho^4} = \frac{2\sqrt{g(\bar{r})}}{1 + \sqrt{g(\bar{r})}} \\ \tilde{H}(\rho) &= 1 - \frac{\omega^4}{\rho^4} = \frac{2}{1 + \sqrt{g(\bar{r})}} \\ F(\rho) &= \left(\frac{H(\rho)}{\tilde{H}(\rho)} \right)^\alpha \cosh^2 \tilde{\theta} - \left(\frac{\tilde{H}(\rho)}{H(\rho)} \right)^\beta \sinh^2 \tilde{\theta} = G(\bar{r})g(\bar{r})^{\frac{\alpha}{2}-1} \end{aligned} \quad (5)$$

where,

$$G(\bar{r}) = 1 + \frac{r_0^4 \cosh^2 \tilde{\theta}}{\bar{r}^4} \quad (6)$$

Note that in writing the last expression for the function $F(\rho)$ in (5) we have used the relation $\alpha + \beta = 2$. This actually fixes one of the independent parameters of the solution and we will be left with only three independent parameters. Now using (5) in the solution (1), we can rewrite the metric and the dilaton as follows,

$$\begin{aligned} ds^2 &= -G(\bar{r})^{-\frac{1}{2}} g(\bar{r})^{\frac{1}{4} + \frac{3\delta_1}{8} + \frac{3\delta_0}{4}} dt^2 + G(\bar{r})^{-\frac{1}{2}} g(\bar{r})^{\frac{1}{4} - \frac{\delta_1}{8} - \frac{\delta_0}{4}} \sum_{i=1}^3 (dx^i)^2 + G(\bar{r})^{\frac{1}{2}} \left(\frac{d\bar{r}^2}{g(\bar{r})} + \bar{r}^2 d\Omega_5^2 \right) \\ e^{2(\phi - \phi_0)} &= g(\bar{r})^{\frac{\delta_1}{4} - 3\delta_0}, \end{aligned} \quad (7)$$

In writing the metric in (7) we have used the condition $\alpha + \beta = 2$ and also the first parameter relation given in (3), $\alpha - \beta = -(3/2)\delta_1$. Combining these two relations we obtain $\alpha = 1 - (3/4)\delta_1$ and this is used to eliminate the parameter α in the above metric. Using this in the second relation given in (3) we obtain a simpler parameter relation as

$$\frac{7}{8}\delta_1^2 + 21\delta_0^2 = 3 \quad (8)$$

Now let us zoom into the region $\bar{r}^4 \sim r_0^4 \ll r_0^4 \cosh^2 \tilde{\theta}$ (note that this implies $\cosh^2 \tilde{\theta} \gg 1$), then the harmonic function $G(\bar{r}) = 1 + r_0^4 \cosh^2 \tilde{\theta} / \bar{r}^4 \approx r_0^4 \cosh^2 \tilde{\theta} / \bar{r}^4 \equiv R^4 / \bar{r}^4$ and the metric in (7) would therefore reduce to,

$$ds^2 = \frac{\bar{r}^2}{R^2} \left(-g(\bar{r})^{\frac{1}{4} + \frac{3\delta_1}{8} + \frac{3\delta_0}{4}} dt^2 + g(\bar{r})^{\frac{1}{4} - \frac{\delta_1}{8} - \frac{\delta_0}{4}} \sum_{i=1}^3 (dx^i)^2 \right) + \frac{R^2}{\bar{r}^2} \left(\frac{d\bar{r}^2}{g(\bar{r})} + \bar{r}^2 d\Omega_5^2 \right) \quad (9)$$

The zooming in here is actually equivalent to taking the parameter $\tilde{\theta} \rightarrow \infty$, keeping the other parameter r_0 finite. This affects the function $G(\bar{r})$ as given above but does not change the form of the function $g(\bar{r})$. The resulting solution (9) is an exact solution of type IIB string theory very similar to the near horizon limit of black D3-brane. Asymptotically the solution (9) has the $\text{AdS}_5 \times \text{S}^5$ structure, but the solution has a curvature singularity at

$\bar{r} = 0$ for the generic values of the parameters δ_0 and δ_1 . The singularity is hidden behind a regular horizon when the parameters take values $\delta_0 = -1/7$ and $\delta_1 = -12/7$ and in that case the solution (9) can be shown to take precisely the form of AdS₅ black hole times S⁵. One can even get a regular solution in the form of bubble of nothing for these values of the parameters by taking a double Wick rotation $t \rightarrow ix^1$ and $x^1 \rightarrow -it$, by making x^1 compact with periodicity $\pi R^2/r_0$. But for other values of the parameters, the singularity cannot be hidden by introducing a blackening factor, as that will not be a string theory solution any more. The black solution is already included in (9) for the particular values of δ_0 and δ_1 as we mentioned and therefore cannot be blackened again. Now changing the coordinate once more by $r = R^2/\bar{r}$, the metric in (9) and the dilaton take the form,

$$ds^2 = \frac{R^2}{r^2} \left(-g(r)^{\frac{1}{4} + \frac{3\delta_1}{8} + \frac{3\delta_0}{4}} dt^2 + g(r)^{\frac{1}{4} - \frac{\delta_1}{8} - \frac{\delta_0}{4}} \sum_{i=1}^3 (dx^i)^2 + \frac{dr^2}{g(r)} \right) + R^2 d\Omega_5^2$$

$$e^{2(\phi-\phi_0)} = g(r)^{\frac{\delta_1}{4} - 3\delta_0}, \quad \text{where, } g(r) = 1 + \frac{r_0^4 r^4}{R^8} \quad (10)$$

The above solution represents the interpolating solution which, as we will show, interpolates between AdS₅ in the UV ($r \rightarrow 0$) and a one parameter family of hyperscaling violating Lifshitz in the IR ($r \rightarrow \infty$). The first part is obvious from (10). When we take $r \rightarrow 0$, the function $g(r) = 1 + r_0^4 r^4 / R^8 \rightarrow 1$, since both r_0 and R are finite. The solution (10) then takes the form,

$$ds^2 = \frac{-dt^2 + \sum_{i=1}^3 (dx^i)^2 + dr^2}{r^2} + d\Omega_5^2$$

$$e^{2(\phi-\phi_0)} = 1, \quad (11)$$

where we have put both R and r_0 to unity for simplicity. Note that there is no parameter dependence in this case although the solution (10) depends on three independent parameters as we mentioned before. Also note that since the radius of S⁵ is constant it gets decoupled and the dilaton is constant. Thus we get AdS₅ in the UV from the solution (10).

Now in the IR, i.e., when we take $r \rightarrow \infty$, the metric and the dilaton in (10) take the form,

$$ds^2 = \frac{1}{r^4} \left(-r^{3+3\gamma} dt^2 + r^{3-\gamma} \sum_{i=1}^3 (dx^i)^2 + \frac{dr^2}{r^2} \right) + d\Omega_5^2$$

$$e^{2(\phi-\phi_0)} = r^{\delta_1 - 12\delta_0} \quad (12)$$

where $\gamma = (1/2)\delta_1 + \delta_0$ and again we have put R and r_0 to unity for simplicity. To show that the above metric has Lifshitz-like scaling symmetry, we need to make another coordinate transformation given by

$$u^2 = r^{\gamma-3} \quad (13)$$

The metric and the dilaton would then be given as,

$$ds^2 = u^{\frac{8}{3-\gamma}} \left(-\frac{dt^2}{u^{\frac{2(3+3\gamma)}{3-\gamma}}} + \frac{\sum_{i=1}^3 (dx^i)^2}{u^2} + \frac{4}{(3-\gamma)^2} \frac{du^2}{u^2} \right) + d\Omega_5^2$$

$$e^{2(\phi-\phi_0)} = u^{-\frac{2(\delta_1-12\delta_0)}{3-\gamma}} \quad (14)$$

It is clear that the part of the metric within bracket given in (14) is invariant under the scaling $t \rightarrow \lambda^z t \equiv \lambda^{(3+3\gamma)/(3-\gamma)} t$, $x^i \rightarrow \lambda x^i$, for $i = 1, 2, 3$ and $u \rightarrow \lambda u$. But the whole metric is not invariant and under the scaling it changes as $ds \rightarrow \lambda^{\theta/d} ds \equiv \lambda^{12/[3(3-\gamma)]} ds$, where $d = 3$ is the spatial dimension of the boundary theory (since S^5 gets decoupled). Thus the dynamical critical exponent z , the hyperscaling violation exponent θ and the spatial dimension d of the theory are

$$z = \frac{3+3\gamma}{3-\gamma}, \quad \theta = \frac{12}{3-\gamma}, \quad d = 3 \quad (15)$$

Since the solution (14) depends on δ_0 and δ_1 , and they are related by the relation (8) we therefore get a one parameter family of hyperscaling violating Lifshitz space-time in the IR ($r \rightarrow \infty$). One can intuitively try to understand why we have a one parameter solution (14). Actually, all the solutions we have either in (10) or in (14) are four parameter solutions as we mentioned. Out of the four we have set $r_0 = R = 1$ and therefore we should be left with two. Now, since (14) can be seen to come from dimensional reduction of ten dimensional type IIB on S^5 , the two parameters must be associated with the two scalars, one coming from the radius of S^5 and the other coming from the ten dimensional dilaton. However, we have fixed the radius of S^5 by putting $\alpha + \beta = 2$, and so we will be left with one parameter γ only as is the case here in (14). Now, as δ_0 and δ_1 are real parameters, it is easy to see from (8) that they cannot take arbitrary values. In fact, δ_0 must lie in the range $0 \leq |\delta_0| \leq \sqrt{1/7} \simeq 0.378$, and δ_1 will automatically fall in the range $0 \leq |\delta_1| \leq \sqrt{24/7} \simeq 1.852$. Now to find the range of z and θ , we must find the range of the parameter $\gamma = (1/2)\delta_1 + \delta_0$. Using the relation (8), it can be checked that the maximum value γ can take is $+1$ and this occurs when $\delta_0 = 1/7$ and $\delta_1 = 12/7$, on the other hand the minimum value γ can take is -1 and in that case $\delta_0 = -1/7$ and $\delta_1 = -12/7$. It is therefore clear that γ can take any real value in the range $-1 \leq \gamma \leq +1$. However, as we will see in the next section that the energy momentum tensor of the boundary theory will further restrict the values of γ to lie in the range $-1 \leq \gamma \leq 0$, otherwise, the energy and the pressure densities of the states associated with the solutions will become negative. This implies that z and θ will lie in the range $0 \leq z \leq 1$ and $3 \leq \theta \leq 4$. Since we have $d = 3$ in this case we therefore have $\theta \geq d$. So, when $z > 0$, such kind of gravity theories ($\theta > d$) have been argued in [10] to be unstable even though the parameters can be shown to satisfy the null energy condition. This may not be completely unexpected. After all, the Lifshitz-like solution is obtained here from non-supersymmetric, singular solution of

string theory. However, we find that the instability argument discussed in [10] either from thermodynamics or from entanglement entropy may not straightforwardly apply here. The reason is, the gravity solutions we have considered are singular and it is not clear how to study thermodynamics and argue about its thermodynamic instability⁷. One might think that since (14) is an exact solution of string theory one can cloak the singularity at $u = 0$ behind a regular horizon by putting a Schwarzschild-like factor $f(u)$ and $f^{-1}(u)$ in front of dt^2 term and du^2 term respectively and thus study the thermodynamics of the system. As we have mentioned earlier, this way of hiding the singularity may be possible, but will not be a solution of string theory. Actually all string theory solutions must be of the form (9) for various allowed values of δ_0 and δ_1 , but it is clear that hyperscaling violating Lifshitz solutions with blackening factor are not contained in it. It is, therefore, not clear how to write a finite temperature version of the solution (14) and obtain the form of entropy (as given in footnote by scaling argument) from there. Also for entanglement entropy, it might be possible to formally obtain an entanglement entropy for this singular system, but it can be shown that it does not satisfy the strong subadditivity condition⁸. So, it is not clear whether such a quantity can be called an entanglement entropy or not. Also we note from the solution (14) that the dilaton is in general not constant and e^ϕ has a power law behavior. So, the dilaton changes as $\phi \rightarrow \phi - (\delta_1 - 12\delta_0)/(3 - \gamma) \log \lambda$ under the scaling we mentioned before. However, when γ takes the two extreme values ± 1 , δ_1 and δ_0 take values $\pm 12/7$ and $\pm 1/7$ respectively, the dilaton becomes a constant.

Thus we have shown that the interpolating solution given in (10) indeed interpolates between AdS_5 in the UV ($r \rightarrow 0$) to a one parameter family of hyperscaling violating Lifshitz in three spatial dimensions in the IR ($r \rightarrow \infty$) with $\theta > d$. We point out that the original solution (1) has four independent parameters as we mentioned before and we fixed one of them by using $\alpha + \beta = 2$, while obtaining the interpolating solution. Out of the remaining three we fixed two more when we used $r_0 = 1$ and $R = 1$ for simplicity. This is the reason we obtained a one parameter family of hyperscaling violating Lifshitz in the IR. In the UV limit, on the other hand, there was no parameter left and we obtained an AdS_5 solution with unit radius.

2.2 Interpretation in the boundary theory : The interpolating solution in the bulk can be interpreted as a flow in the boundary gauge theory. The dependence of the bulk field on the inverse radial coordinate r is interpreted as the dependence on the energy parameter in the boundary theory. In particular, the dilaton being non-constant (except for $\gamma = \pm 1$)

⁷On the other hand, a simple scaling argument for the Lifshitz theory with hyperscaling violation shows that the thermal entropy of such theory must scale as $S \sim T^{(d-\theta)/z}$ and so the specific heat for $\theta > d$ must be negative indicating a thermal instability. We thank Sandip Trivedi for pointing this out to us.

⁸This is also true for any thermodynamically unstable system as discussed in [35].

implies the running of the 't Hooft coupling⁹ since the dilaton couples to the dimension four operator $\text{Tr}(F^2)$ in the gauge theory. The dilaton approaching asymptotically ($r \rightarrow 0$) to a constant value, $\phi - \phi_0 \sim r^4$, can be interpreted as the flow of the gauge theory to a UV-stable fixed point [20, 22]. In this case, as we have seen, the geometry (10) reduces to AdS_5 (11) and the corresponding gauge theory at the fixed point is $\mathcal{N} = 4$, $D = 4$ super Yang-Mills theory. The solution (10) away from $r = 0$ can be regarded as a deformation of AdS_5 , by the addition of various normalizable modes like the dilaton and the various components of the metric. Note that although these additions preserve the original $\text{SO}(6)$ invariance, they do not preserve the $\text{SO}(1,3)$ Lorentz invariance of the metric (since the coefficient of g_{tt} is different from those of g_{ii} 's, where $i = 1, 2, 3$). They also break the conformal invariance as well as supersymmetry [20, 22]. In terms of the boundary gauge theory, we can think of the additions of these terms as the deformation of the $\mathcal{N} = 4$, $D = 4$ super Yang-Mills theory. The additions of various normalizable modes amounts to the turning on the expectation values of various gauge invariant operators [36]. So, for example, the addition of dilaton gives an expectation value $\langle \text{Tr}(F^2) \rangle \sim (\delta_1 - 12\delta_0)$ in terms of the dimensionless parameters of the solution, where δ_1 and δ_0 are related by the relation (8). Note that the expectation value is dimensionless and that is because we have set the dimensionful parameters r_0 and R to unity. Also, the addition of the metric components give expectation values of an energy-momentum tensor [21] in the form $\langle T_{\mu\nu} \rangle \sim \text{diag}(-3\gamma, -\gamma, -\gamma, -\gamma)$, where $\mu, \nu = 0, 1, 2, 3$. This energy-momentum tensor¹⁰ can be regarded as arising due to the excitations above the ground state representing zero temperature $\mathcal{N} = 4$, $D = 4$ super Yang-Mills theory. Now since the sign of the energy momentum tensor depends on the sign of γ , it is clear that the states with $\gamma > 0$ are unphysical since that will give negative energy and pressure densities¹¹. From the form of the energy momentum tensor given above it is clear that it breaks the Lorentz invariance of the theory and as the boundary theory flows to IR ($r \rightarrow \infty$), we end up with a fixed point having a non-relativistic symmetry of the form of Lifshitz scaling symmetry with

⁹Similar behavior has been noted in some different non-supersymmetric type IIB solution in [20, 22].

¹⁰Similar form of energy momentum tensors are given in [21, 22]. Actually the solution we have discussed in section 2.1 is an isotropic version of the solution discussed in [22]. In fact they can be mapped to each other by a coordinate transformation when the solution of [22] is made isotropic. Also we have fixed the value of the volume scalar of the 5-sphere by a special choice of some of our parameters just for simplification but it is not necessary.

¹¹However, we mention that the negative energy solutions can be obtained from (10) if we make a Wick rotation $t \rightarrow ix^1$ and $x^1 \rightarrow -it$ and compactify x^1 . This way we get new solutions which breaks $\text{SO}(3)$ symmetry down to $\text{SO}(2)$ and the expectation value of the energy momentum tensor for these new solutions would be given as $\langle T_{\mu\nu} \rangle \sim \text{diag}(+\gamma, +3\gamma, -\gamma, -\gamma)$. Now since $\gamma \leq 0$ are physical we see that the new solutions (with $\gamma < 0$) are negative energy solutions. $\gamma = -1$ is special since in this case the solution can be made regular if x^1 is made compact with periodicity $\pi R^2/r_0$ as we mentioned earlier. This solution is called AdS 'bubble of nothing' or AdS soliton. In the boundary theory this negative energy manifests itself in the form of Casimir energy as shown in [21].

hyperscaling violation. Note that the above energy-momentum tensor is actually traceless, i.e., $\eta^{\mu\nu}\langle T_{\mu\nu}\rangle = 0$ as it should be. From the form of energy-momentum tensor it is clear that when $\gamma = 0$, it vanishes, indicative of the restoration of Lorentz or relativistic symmetry [22]. Indeed we find from (15) that when $\gamma = 0$, the dynamical critical exponent $z = 1$ and we recover relativistic symmetry. This happens when $\delta_1 = \pm 2\sqrt{6}/7$ and $\delta_0 = \mp\sqrt{6}/7$. Note that when $\gamma = 0$, energy density vanishes which means that this is a zero energy state. Moreover, since this state has non-zero VEV of $\text{Tr}(F^2)$, supersymmetry is completely broken. So, this is very much like a QCD state at zero temperature. In fact, when $\gamma = 0$, the solution (10) can be mapped exactly to eq.(2.1), (2.2) of [22] (for $\delta = 2$ there) and eq.(3.10), (3.11) of [37]. The boundary theory with various of its properties, resembling those of QCD, have been discussed at length in both these references. We would like to point out that in our study of flows starting from $\mathcal{N} = 4$, $D = 4$ super Yang-Mills theory, the QCD-like theory for $\gamma = 0$ appears as an intermediate state in the IR and in the deep IR we get a theory which is conformal to $\text{AdS}_5 \times S^5$. Also, we have seen that when $\gamma = -1$, the dilaton remains constant and in those cases we do not turn on $\langle\text{Tr}(F^2)\rangle$ in the gauge theory. From the metric in (10) it can be checked that when $\gamma = -1$, i.e., when $\delta_1 = -12/7$ and $\delta_0 = -1/7$, the metric reduces precisely to the AdS_5 black hole. The boundary theory then represents $\mathcal{N} = 4$, $D = 4$ super Yang-Mills theory at finite temperature. For the generic values of γ , the solution given in (10) can be mapped to the solution discussed in [38]. The boundary theory in this case has been interpreted as a finite temperature QCD like theory, some whose properties have been discussed there. Again, we point out that this theory appears only in the intermediate stage and in the deep IR we get Lifshitz theories with hyperscaling violation as discussed in previous section.

We, therefore, conclude that the interpolating solution described in section 2.1 can be interpreted as a flow in the boundary theory for all values of γ in the range $-1 \leq \gamma \leq 0$. Starting from $\mathcal{N} = 4$, $D = 4$ super Yang-Mills theory in the UV, we end up with a one-parameter (given by γ) family of non-supersymmetric Lifshitz theory with hyperscaling violation in the IR (this happens only for $-1 < \gamma < 0$), where the various exponents are given in (15). For $\gamma = 0$, the IR theory still remains relativistic although the supersymmetry is broken by the non-zero VEV of $\text{Tr}(F^2)$. In fact the gravity dual in this case can be seen from (14) to take the form of conformal AdS_5 times S^5 . On the other hand, for $\gamma = -1$, the theory flows to a finite temperature Yang-Mills theory. Here again the supersymmetry is broken but we have $\langle\text{Tr}(F^2)\rangle = 0$ and $\langle T_{\mu\nu}\rangle \neq 0$. For all other values of γ , we have a flow to hyperscaling violating Lifshitz with $\theta > d$ in the IR. Like in the previous cases we always have broken supersymmetry and the flows are triggered by non-zero VEV of $\text{Tr}(F^2)$ and the energy momentum tensor. Even though the final state is hyperscaling violating Lifshitz with $\theta > d$, but as we have pointed out that this criterion may not apply in the present context of non-supersymmetric solutions of string theory to conclude that the gravity solution is

unstable invalidating the existence of the flow in the boundary theory.

2.3 Interpolation from supersymmetric solution : In subsection 2.1 we obtained a solution which interpolates between AdS_5 in the UV and a one parameter family of hyperscaling violating $d = 3$ Lifshitz solution in the IR from certain non-supersymmetric solution of type IIB string theory. The Lifshitz solution we obtained in that case has $\theta > d$ and might be unstable. In this subsection we obtain another similar interpolating solution from some supersymmetric bound state solution (F-D2) of type IIA string theory and in this case the Lifshitz solution has $\theta = d - 1$ as was noted before in [39].

The quarter supersymmetric intersecting F-D p bound state solutions of type II string theory are given in eq.(2.6) of ref.[39] and were obtained from the standard D1-D5 solution of type IIB string theory by applying a series of T- as well as S-duality symmetries. For the purpose of constructing the supersymmetric interpolating solution we will take F-D2 solution of type IIA string theory which can be written from eq.(2.6) of ref.[39] for $p = 2$ and has the following form,

$$ds^2 = H_2^{\frac{1}{2}} \left[-H_1^{-1} H_2^{-1} dt^2 + H_2^{-1} \sum_{i=1}^2 (dx^i)^2 + H_1^{-1} (dx^3)^2 + d\bar{r}^2 + \bar{r}^2 d\Omega_5^2 \right]$$

$$e^{2(\phi-\phi_0)} = \frac{H_2^{\frac{1}{2}}}{H_1}, \quad B_{[2]} = (1 - H_1^{-1}) dt \wedge dx^3, \quad A_{[3]} = (1 - H_2^{-1}) dt \wedge dx^1 \wedge dx^2 \quad (16)$$

Here $H_{1,2}$ are two harmonic functions and are given as,

$$H_{1,2} = 1 + \frac{Q_{1,2}}{\bar{r}^4} \quad (17)$$

with $Q_{1,2}$ representing the charges associated with F-strings and D2-branes respectively. $B_{[2]}$ and $A_{[3]}$ are respectively the NSNS 2-form and RR 3-form which couple to F-strings and D2-branes. It is clear from the solution above that F-strings lie along x^3 , whereas D2-branes lie along x^1 and x^2 . Note that here F-strings are delocalized along x^1 and x^2 and D2-branes are delocalized along x^3 and they together form a (3+1)-dimensional world-volume. Let us now zoom into the region,

$$\bar{r}^4 \sim Q_1 \ll Q_2. \quad (18)$$

Then, the harmonic function H_2 can be approximated as $H_2 \approx Q_2/\bar{r}^4$, whereas, H_1 remains the same. Therefore, the metric and the other form-fields take the form,

$$ds^2 = \frac{Q_2^{\frac{1}{2}}}{\bar{r}^2} \left[-\frac{\bar{r}^4}{Q_2} H_1^{-1} dt^2 + \frac{\bar{r}^4}{Q_2} \sum_{i=1}^2 (dx^i)^2 + H_1^{-1} (dx^3)^2 + d\bar{r}^2 + \bar{r}^2 d\Omega_5^2 \right]$$

$$e^{2(\phi-\phi_0)} = \frac{Q_2^{\frac{1}{2}}}{\bar{r}^2 H_1}, \quad B_{[2]} = (1 - H_1^{-1}) dt \wedge dx^3, \quad A_{[3]} = -\frac{\bar{r}^4}{Q_2} dt \wedge dx^1 \wedge dx^2 \quad (19)$$

Note that since the solution is no longer asymptotically flat we have dropped a constant in $A_{[3]}$. Now changing the coordinate by $r = Q_2^{\frac{1}{2}}/\bar{r}$, the solution (19) reduces to,

$$ds^2 = \frac{Q_2^{\frac{1}{2}}}{r^2} \left[-H_1^{-1} dt^2 + \sum_{i=1}^2 (dx^i)^2 + dr^2 \right] + \frac{r^2}{Q_2^{\frac{1}{2}}} H_1^{-1} (dx^3)^2 + Q_2^{\frac{1}{2}} d\Omega_5^2$$

$$e^{2(\phi-\phi_0)} = \frac{r^2}{Q_2^{\frac{1}{2}}} H_1^{-1}, \quad B_{[2]} = (1 - H_1^{-1}) dt \wedge dx^3, \quad A_{[3]} = -\frac{Q_2}{r^4} dt \wedge dx^1 \wedge dx^2 \quad (20)$$

where $H_1 = 1 + (Q_1/Q_2^2)r^4$. Eq.(20) is the required interpolating solution¹² and we will show how this interpolates between AdS_5 in the UV ($r \rightarrow 0$) and a hyperscaling violating Lifshitz with $d = 3$, $z = 3$ and $\theta = 2 = d - 1$ in the IR ($r \rightarrow \infty$). It is clear from (20) that in the UV, as $r \rightarrow 0$, $H_1 \rightarrow 1$ and therefore the solution reduces to

$$ds^2 = \frac{Q_2^{\frac{1}{2}}}{r^2} \left[-dt^2 + \sum_{i=1}^2 (dx^i)^2 + dr^2 \right] + \frac{r^2}{Q_2^{\frac{1}{2}}} (dx^3)^2 + Q_2^{\frac{1}{2}} d\Omega_5^2$$

$$e^{2(\phi-\phi_0)} = \frac{r^2}{Q_2^{\frac{1}{2}}}, \quad B_{[2]} = 0, \quad A_{[3]} = -\frac{Q_2}{r^4} dt \wedge dx^1 \wedge dx^2 \quad (21)$$

The metric has the structure $\text{AdS}_4 \times \mathbb{R} \times S^5$, where, \mathbb{R} corresponds to x^3 direction and becomes small as $r \rightarrow 0$. In that case we can compactify x^3 direction and end up with a nine dimensional solution where the metric will have $\text{AdS}_4 \times S^5$ structure (given above without $(dx^3)^2$ term) and the nine dimensional dilaton will have the form $e^{2(\hat{\phi}-\hat{\phi}_0)} = r/Q_2^{1/4}$ with the 3-form remaining the same. However, if we want to remain in ten dimensions, the above supergravity description is not good since in the UV, the radius along the compact x^3 direction can become of the order of string length and the stringy effects will be important. The good description would be given in the T-dual frame, i.e., if we take T-duality along x^3 direction. It can be checked by taking T-duality along x^3 that, the metric in (21) reduces precisely to $\text{AdS}_5 \times S^5$ and the dilaton becomes constant. The 3-form becomes a 4-form, whose field-strength must be made self-dual. This is exactly the near horizon structure of large number of coincident D3-brane configuration. This way in the UV we recover the AdS_5 structure.

On the other hand, in the IR when $r \rightarrow \infty$, the harmonic function $H_1 \approx (Q_1/Q_2^2)r^4$ and therefore, the interpolating solution (20) reduces to

$$ds^2 = Q_2^{\frac{3}{2}} \left[-\frac{Q_2}{Q_1} \frac{dt^2}{r^6} + \frac{\sum_{i=1}^2 (dx^i)^2 + dr^2}{Q_2 r^2} + \frac{(dx^3)^2}{Q_1 r^2} \right] + Q_2^{\frac{1}{2}} d\Omega_5^2$$

$$e^{2(\phi-\phi_0)} = \frac{Q_2^{\frac{3}{2}}}{Q_1 r^2}, \quad B_{[2]} = -\frac{Q_2^2}{Q_1 r^4} dt \wedge dx^3, \quad A_{[3]} = -\frac{Q_2}{r^4} dt \wedge dx^1 \wedge dx^2 \quad (22)$$

¹²A T-dual version of this solution has been given in [40] but the interpolation there is quite different from that given here.

This configuration was obtained in [39] in eq.(2.11) with $p = 2$. However, note that to match exactly with the solution given in [39], we have to replace r by $Q_2^{\frac{1}{2}}r$ because of slightly different notation we choose here. In order to see the scaling symmetry we dimensionally reduce the metric to five dimensions and express the resulting metric in Einstein frame. It has the form,

$$ds^2 = Q_1^{\frac{2}{3}} Q_2^{\frac{5}{3}} r^{\frac{4}{3}} \left[-\frac{Q_2}{Q_1} \frac{dt^2}{r^6} + \frac{\sum_{i=1}^2 (dx^i)^2 + dr^2}{Q_2 r^2} + \frac{(dx^3)^2}{Q_1 r^2} \right] \quad (23)$$

Now from (23) we observe that under the scaling $t \rightarrow \lambda^3 t$, $x^{1,2,3} \rightarrow \lambda x^{1,2,3}$ and $r \rightarrow \lambda r$, the part of the metric in the bracket is invariant, but the full metric is not. In fact, under the above scaling the metric changes as $ds \rightarrow \lambda^{2/3} ds \equiv \lambda^{\theta/d} ds$, where θ is the hyperscaling violation exponent. We thus conclude that in the IR, the interpolating solution gives a hyperscaling violating Lifshitz space-time with spatial dimension $d = 3$, dynamical exponent $z = 3$ and hyperscaling violating exponent $\theta = 2 = d - 1$.

It is, therefore, clear that the solution (20) indeed interpolates between AdS_5 in the UV and a hyperscaling violating Lifshitz with $z = 3$, $d = 3$ and $\theta = 2$ in the IR. Note that although the interpolating solution (20) belongs to type IIA string theory, in the UV its gravity description is given by $\text{AdS}_5 \times S^5$ which is a solution of type IIB string theory. This is because the metric in (20) does not have a good gravity description in the UV if we want to remain in ten space-time dimensions, and in that case the only gravity description be given in the T-dual frame, as we have argued, which takes the solution to type IIB theory. But the problem in this case is that because of the T-duality transformation, it is difficult to give the interpretation of this interpolation as a flow in the boundary theory, nevertheless, it will certainly be interesting to have a better understanding of this issue.

3. Conclusion : To summarize, in this paper we have constructed two type II string theory solutions which interpolate between an AdS_5 metric in the UV and a hyperscaling violating Lifshitz metric in the IR. Our starting point in the first case was a known non-supersymmetric D3-brane intersecting with a chargeless D0-brane solution of type IIB string theory. This is a four parameter solution which gives the interpolating solution when we expressed it in a suitable coordinate and restricted one of the parameters to a special value. The interpolating solution gives AdS_5 metric in the UV (when $r \rightarrow 0$) and a one parameter family of Lifshitz metric (in three spatial dimension) with hyperscaling violation in the IR (when $r \rightarrow \infty$). The transverse 5-dimensional space which is a 5-sphere gets completely decoupled in the solution. The IR Lifshitz-like metric has a dynamical critical exponent $z = (3 + 3\gamma)/(3 - \gamma)$ and a hyperscaling violation exponent $\theta = 12/(3 - \gamma)$, where γ is a real parameter and lies in the range $-1 \leq \gamma \leq 1$. From physical arguments we have shown that γ must be restricted to lie in the range $-1 \leq \gamma \leq 0$ only. We, therefore, have z lying in the range $0 \leq z \leq 1$ and θ lying in the range $3 \leq \theta \leq 4$. When $\gamma = -1$, the dilaton is constant, otherwise dilaton is

a function of inverse radial coordinate r . When $\gamma = -1$, the interpolating solution reduces to the AdS_5 black hole. On the other hand, when $\gamma = 0$, the Lorentz invariance is restored and in that case the IR theory is not non-relativistic or Lifshitz-like, rather it is conformal to AdS_5 , i.e., relativistic. We have noted that since in the IR we have Lifshitz-like theory with $d = 3$ and $3 \leq \theta \leq 4$, we have $\theta \geq d$. These theories (for $\theta > d$ and $z > 0$) are usually unstable [10]. However, we have mentioned that the argument of Dong et. al. [10] may not be as straightforward as we think. This is because the solution itself is singular and it is not clear how to study thermodynamics and how to define entanglement entropy for the system with $\theta > d$. We have seen that the interpolation of the supergravity solution can be understood in the boundary theory as a flow, where on physical grounds we have excluded the positive values of γ . By this flow $\mathcal{N} = 4$, $D = 4$ super Yang-Mills theory in the ultraviolet goes over to an infrared fixed point which is non-relativistic and given by the strongly interacting many body system with a Lifshitz-like (with hyperscaling violation) scaling symmetry. This flow is triggered by turning on the expectation values of the operator $\text{Tr}(F^2)$ and various components of an energy-momentum tensor $T_{\mu\nu}$. They break conformal invariance, supersymmetry and $\text{SO}(1,3)$ Lorentz invariance of the UV theory.

In the second case we consider a quarter supersymmetric F-D2 bound state solution of type IIA string theory. By zooming into certain region of space in this solution we have shown how to obtain an interpolating solution which interpolates between AdS_5 in the UV and a hyperscaling violating Lifshitz in $d = 3$ with $z = 3$ and $\theta = 2$. Thus in the supersymmetric case, we indeed find a hyperscaling violating Lifshitz which has $\theta < d$, in fact, it has $\theta = d - 1$ in contrast to the non-supersymmetric case where we found that the hyperscaling violating Lifshitz has $\theta > d$. However, in this case the interpolation involves a T-duality transformation and that is the reason it is difficult to interpret this interpolation in terms of certain RG flow in the boundary theory. It would be nice to have a better understanding of this issue and we leave it for the future.

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References

- [1] J. M. Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998) [hep-th/9711200]; E. Witten, Adv. Theor. Math. Phys. **2**, 253 (1998) [hep-th/9802150]; S. S. Gubser, I. R. Klebanov

- and A. M. Polyakov, Phys. Lett. B **428**, 105 (1998) [hep-th/9802109].
- [2] O. Aharony, S. S. Gubser, J. M. Maldacena, H. Ooguri and Y. Oz, Phys. Rept. **323**, 183 (2000) [hep-th/9905111].
- [3] J. Casalderrey-Solana, H. Liu, D. Mateos, K. Rajagopal and U. A. Wiedemann, arXiv:1101.0618 [hep-th].
- [4] S. A. Hartnoll, Class. Quant. Grav. **26**, 224002 (2009) [arXiv:0903.3246 [hep-th]]; J. McGreevy, Adv. High Energy Phys. **2010**, 723105 (2010) [arXiv:0909.0518 [hep-th]]; S. Sachdev, Lect. Notes Phys. **828**, 273 (2011) [arXiv:1002.2947 [hep-th]]; S. Sachdev, Ann. Rev. Condensed Matter Phys. **3**, 9 (2012) [arXiv:1108.1197 [cond-mat.str-el]].
- [5] S. Kachru, X. Liu and M. Mulligan, Phys. Rev. D **78**, 106005 (2008) [arXiv:0808.1725 [hep-th]].
- [6] M. Taylor, arXiv:0812.0530 [hep-th].
- [7] S. A. Hartnoll, J. Polchinski, E. Silverstein and D. Tong, JHEP **1004**, 120 (2010) [arXiv:0912.1061 [hep-th]]; K. Balasubramanian and K. Narayan, JHEP **1008**, 014 (2010) [arXiv:1005.3291 [hep-th]]; A. Donos and J. P. Gauntlett, JHEP **1012**, 002 (2010) [arXiv:1008.2062 [hep-th]]; A. Donos, J. P. Gauntlett, N. Kim and O. Varela, JHEP **1012**, 003 (2010) [arXiv:1009.3805 [hep-th]]; H. Singh, JHEP **1012**, 061 (2010) [arXiv:1009.0651 [hep-th]]; R. Gregory, S. L. Parameswaran, G. Tasinato and I. Zavala, JHEP **1012**, 047 (2010) [arXiv:1009.3445 [hep-th]]; D. Cassani and A. F. Faedo, JHEP **1105**, 013 (2011) [arXiv:1102.5344 [hep-th]]; K. Narayan, Phys. Rev. D **84**, 086001 (2011) [arXiv:1103.1279 [hep-th]]; N. Halmagyi, M. Petrini and A. Zaffaroni, JHEP **1108**, 041 (2011) [arXiv:1102.5740 [hep-th]]; W. Chemissany and J. Hartong, Class. Quant. Grav. **28**, 195011 (2011) [arXiv:1105.0612 [hep-th]]; B. Gouteraux and E. Kiritsis, JHEP **1112**, 036 (2011) [arXiv:1107.2116 [hep-th]].
- [8] D. S. Fisher, Phys. Rev. Lett. **56**, 416 (1986).
- [9] C. Charmousis, B. Gouteraux, B. S. Kim, E. Kiritsis and R. Meyer, JHEP **1011**, 151 (2010) [arXiv:1005.4690 [hep-th]].
- [10] X. Dong, S. Harrison, S. Kachru, G. Torroba and H. Wang, JHEP **1206**, 041 (2012) [arXiv:1201.1905 [hep-th]].
- [11] N. Ogawa, T. Takayanagi and T. Ugajin, JHEP **1201**, 125 (2012) [arXiv:1111.1023 [hep-th]].

- [12] L. Huijse, S. Sachdev and B. Swingle, Phys. Rev. B **85**, 035121 (2012) [arXiv:1112.0573 [cond-mat.str-el]].
- [13] P. Dey and S. Roy, JHEP **1206**, 129 (2012) [arXiv:1203.5381 [hep-th]]; P. Dey and S. Roy, Phys. Rev. D **86**, 066009 (2012) [arXiv:1204.4858 [hep-th]]; P. Dey and S. Roy, Phys. Lett. B **720**, 419 (2013) [arXiv:1209.1049 [hep-th]]; P. Dey and S. Roy, JHEP **1311**, 113 (2013) [arXiv:1306.1071 [hep-th]].
- [14] K. Narayan, Phys. Rev. D **85**, 106006 (2012) [arXiv:1202.5935 [hep-th]]; H. Singh, JHEP **1207**, 082 (2012) [arXiv:1202.6533 [hep-th]]; E. Perlmutter, JHEP **1206**, 165 (2012) [arXiv:1205.0242 [hep-th]]; M. Alishahiha, E. O Colgain and H. Yavartanoo, JHEP **1211**, 137 (2012) [arXiv:1209.3946 [hep-th]]; J. Gath, J. Hartong, R. Monteiro and N. A. Obers, JHEP **1304**, 159 (2013) [arXiv:1212.3263 [hep-th]].
- [15] B. Zhou and C. J. Zhu, hep-th/9905146.
- [16] V. D. Ivashchuk and V. N. Melnikov, Class. Quant. Grav. **18**, R87 (2001) [hep-th/0110274]; Y. G. Miao and N. Ohta, Phys. Lett. B **594**, 218 (2004) [hep-th/0404082]; J. X. Lu and S. Roy, JHEP **0502**, 001 (2005) [hep-th/0408242].
- [17] P. Brax, G. Mandal and Y. Oz, Phys. Rev. D **63**, 064008 [hep-th/0005242]; U. H. Danielsson, A. Guijosa and M. Kruczenski, JHEP **0109**, 011 (2001) [hep-th/0106201]; J. X. Lu and S. Roy, Phys. Lett. B **599**, 313 (2004) [hep-th/0403147]; J. X. Lu and S. Roy, JHEP **0411**, 008 (2004) [hep-th/0409019]; J. X. Lu and S. Roy, JHEP **0506**, 026 (2005) [hep-th/0503007].
- [18] J. X. Lu, S. Roy, Z. -L. Wang and R. -J. Wu, Nucl. Phys. B **813**, 259 (2009) [arXiv:0710.5233 [hep-th]].
- [19] J. G. Russo, Phys. Lett. B **435**, 284 (1998) [hep-th/9804209]; Y. Kiem and D. Park, Phys. Rev D **59**, 044010 (1999) [hep-th/9809174];
- [20] A. Kehagias and K. Sfetsos, Phys. Lett. B **454**, 270 (1999) [hep-th/9902125]; S. S. Gubser, hep-th/9902155; S. Nojiri and S. D. Odintsov, Phys. Lett. B **458**, 226 (1999) [hep-th/9904036].
- [21] R. C. Myers, Phys. Rev. D **60**, 046002 (1999) [hep-th/9903203].
- [22] N. R. Constable and R. C. Myers, JHEP **9911**, 020 (1999) [hep-th/9905081].
- [23] R. Gregory and R. Laflamme, Phys. Rev. Lett. **70**, 2837 (1993) [hep-th/9301052]; R. Gregory and R. Laflamme, Nucl. Phys. B **428**, 399 (1994) [hep-th/9404071].

- [24] S. S. Gubser and I. Mitra, hep-th/0009126; S. S. Gubser and I. Mitra, JHEP **0108**, 018 (2001) [hep-th/0011127].
- [25] T. Azeyanagi, W. Li and T. Takayanagi, JHEP **0906**, 084 (2009) [arXiv:0905.0688 [hep-th]].
- [26] D. Mateos and D. Trancanelli, Phys. Rev. Lett. **107**, 101601 (2011) [arXiv:1105.3472 [hep-th]]; D. Mateos and D. Trancanelli, JHEP **1107**, 054 (2011) [arXiv:1106.1637 [hep-th]].
- [27] H. Braviner, R. Gregory and S. F. Ross, Class. Quant. Grav. **28**, 225028 (2011) [arXiv:1108.3067 [hep-th]].
- [28] S. P. Kumar, JHEP **1208**, 155 (2012) [arXiv:1206.5140 [hep-th]].
- [29] H. Singh, JHEP **1308**, 097 (2013) [arXiv:1305.3784 [hep-th]].
- [30] A. F. Faedo, B. Fraser and S. P. Kumar, JHEP **1402**, 066 (2014) [arXiv:1310.0206 [hep-th]].
- [31] S. Kachru, N. Kundu, A. Saha, R. Samanta and S. P. Trivedi, JHEP **1403**, 074 (2014) [arXiv:1310.5740 [hep-th]].
- [32] M. Ghodrati, arXiv:1404.5399 [hep-th].
- [33] H. Bai, J. X. Lu and S. Roy, JHEP **0701**, 094 (2007) [hep-th/0610264].
- [34] S. Roy, Phys. Rev. D **89** 104044 (2014) [arXiv:1402.2912 [hep-th]].
- [35] J. Bhattacharya, M. Nozaki, T. Takayanagi and T. Ugajin, Phys. Rev. Lett. **110**, no. 9, 091602 (2013) [arXiv:1212.1164].
- [36] V. Balasubramanian, P. Kraus and A. E. Lawrence, Phys. Rev. D **59**, 046003 (1999) [hep-th/9805171]; V. Balasubramanian, P. Kraus, A. E. Lawrence and S. P. Trivedi, Phys. Rev. D **59**, 104021 (1999) [hep-th/9808017]. U. H. Danielsson, E. Keski-Vakkuri and M. Kruczenski, JHEP **9901**, 002 (1999) [hep-th/9812007].
- [37] C. Csaki and M. Reece, JHEP **0705**, 062 (2007) [hep-ph/0608266].
- [38] Y. Kim, B. H. Lee, C. Park and S. J. Sin, JHEP **0709**, 105 (2007) [hep-th/0702131].
- [39] The first reference in [13].
- [40] H. Singh, JHEP **1104**, 118 (2011) [arXiv:1011.6221 [hep-th]].